

Undetermined Coefficients for $y'' + p(x)y' + q(x)y = g(x)$

$g(x)$	$y_p(x)$
(1) $p_n(x) = a_n x^n + \dots + a_1 x + a_0$	$x^s P_n(x) = x^s (A_n x^n + \dots + A_1 x + A_0)$
(2) $a e^{\alpha x}$	$x^s A e^{\alpha x}$
(3) $a \cos \beta x + b \sin \beta x$	$x^s (A \cos \beta x + B \sin \beta x)$
(4) $p_n(x) e^{\alpha x}$	$x^s P_n(x) e^{\alpha x}$
(5) $p_n(x) \cos \beta x + q_m(x) \sin \beta x$, where $q_m(x) = b_m x^m + \dots + b_1 x + b_0$	$x^s \{P_N(x) \cos \beta x + Q_N(x) \sin \beta x\}$, where $Q_N(x) = B_n x^N + \dots + B_1 x + B_0$ and $N = \max(n, m)$
(6) $a e^{\alpha x} \cos \beta x + b e^{\alpha x} \sin \beta x$	$x^s (A e^{\alpha x} \cos \beta x + B e^{\alpha x} \sin \beta x)$
(7) $p_n(x) e^{\alpha x} \cos \beta x + q_m(x) e^{\alpha x} \sin \beta x$	$x^s e^{\alpha x} \{P_N(x) \cos \beta x + Q_N(x) \sin \beta x\}$, where $N = \max(n, m)$

The nonnegative integer s is chosen to be the least integer such that no term in $y_p(x)$ is a solution of the corresponding homogeneous equation $y'' + p(x)y' + q(x)y = 0$.

Variation of Parameters

If y_1 and y_2 are two linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$, then a particular solution of $y'' + p(x)y' + q(x)y = g(x)$ is $y = v_1 y_1 + v_2 y_2$, where

$$v_1(x) = \int \frac{-g(x)y_2(x)}{W[y_1, y_2](x)} dx, \quad v_2(x) = \int \frac{g(x)y_1(x)}{W[y_1, y_2](x)} dx,$$

and $W[y_1, y_2](x) = y_1(x)y_2'(x) - y_1'(x)y_2(x)$.